Quantization of Energy

Failures of Classical mechanics

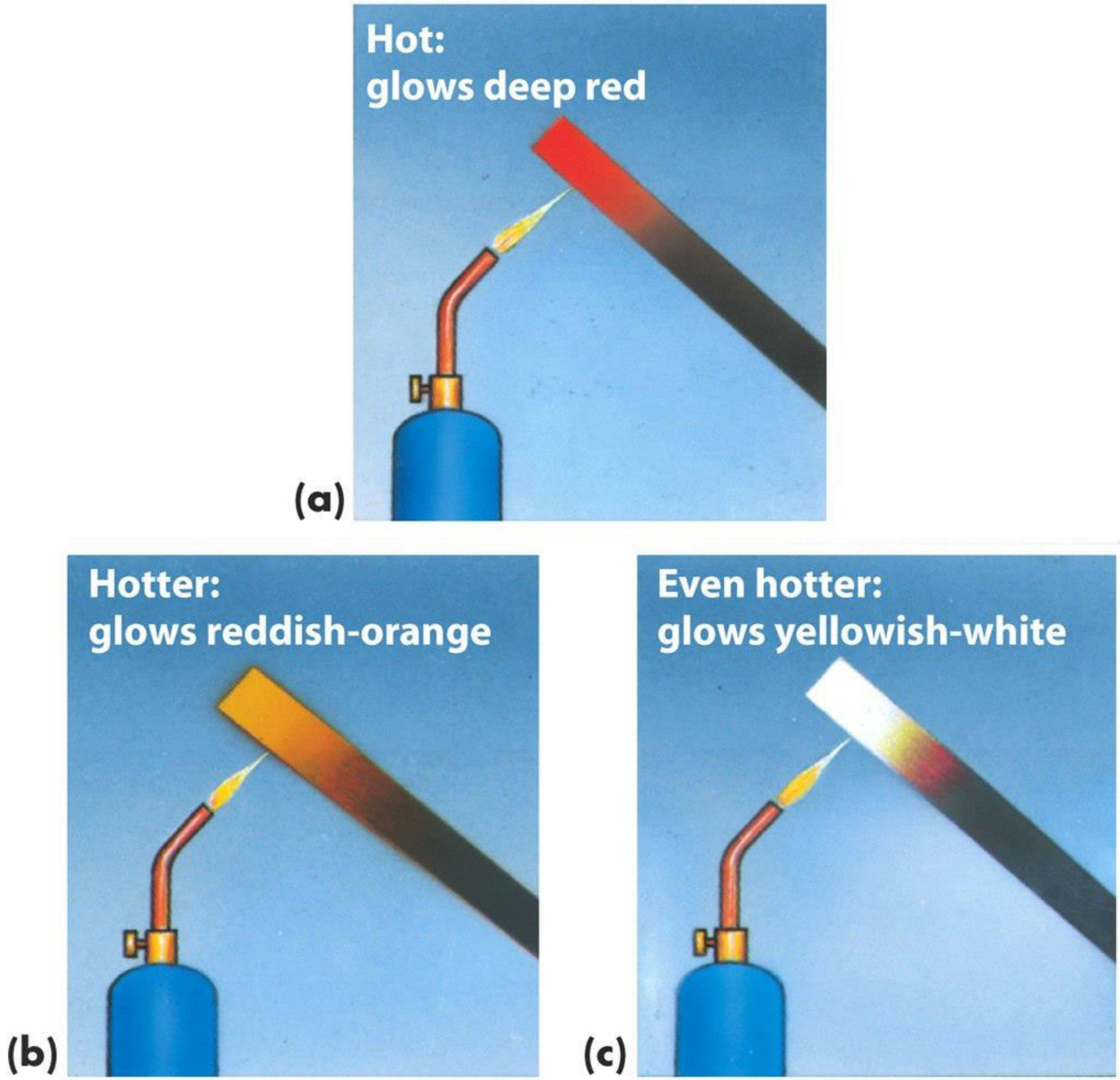
➢Black Body Radiation:

WHAT IS BLACK BODY ?

A **black body** is a theoretical object that **absorbs 100% of the radiation** that hits it. Therefore it reflects no radiation and appears perfectly black.

It is also a **perfect emitter of radiation**. At a particular temperature the black body would emit the maximum amount of energy possible for that temperature.

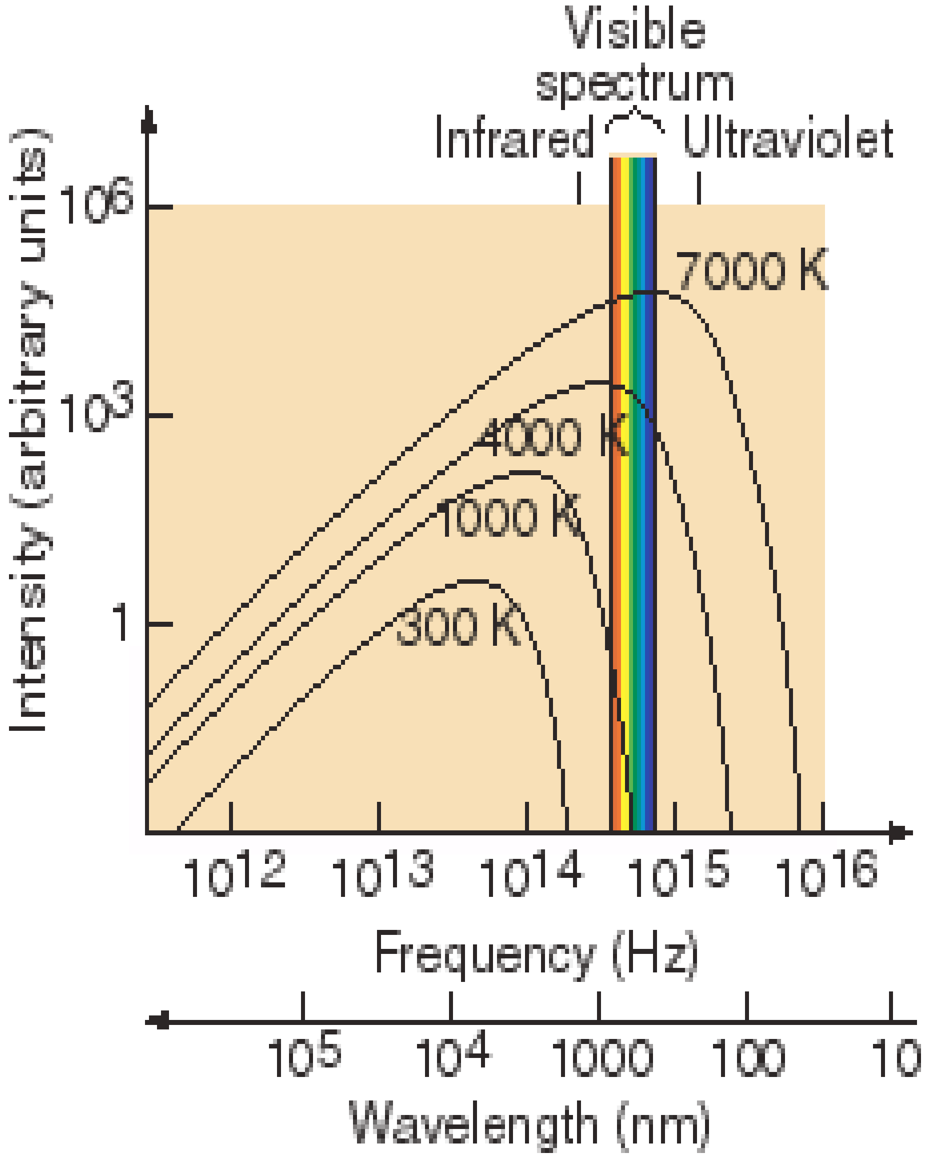
An opaque object emits electromagnetic radiation according to its temperature



- Red stars are relatively cool. A yellow star, such as our own sun, is hotter. A blue star is very hot.

Planck Spectrum

- **As an object is heated, the radiation it emits peaks at higher and higher frequencies.**
- **Shown here are curves corresponding to temperatures of**
  - **300 K (room temperature),**
  - **1000 K (begin to glow deep red)**
  - **4000 K (red hot), and**
  - **7000 K (white hot).**



Rayleigh-Jean's law

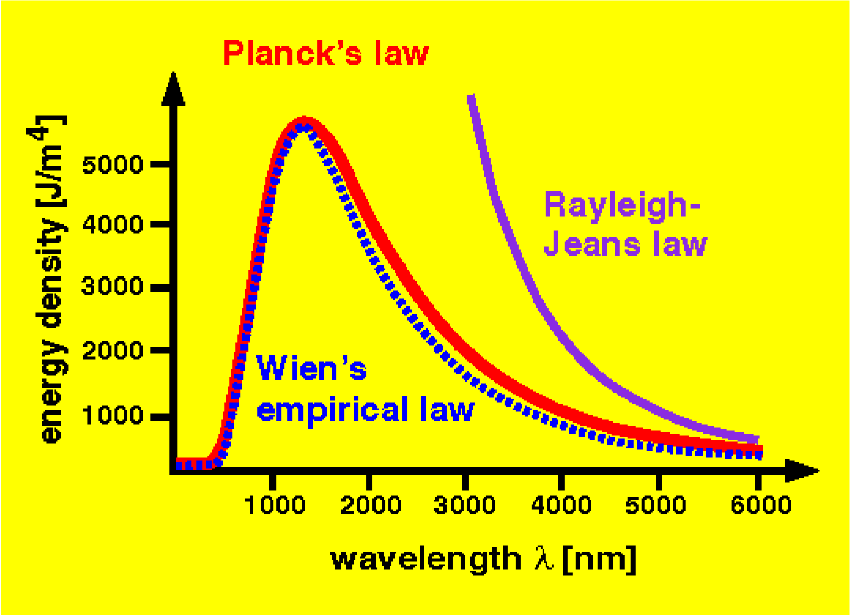
- **The energy density p_v per unit frequency interval at a frequency v is, according to the The Rayleigh-Jeans Radiation,**

$$\rho_v = \frac{8\pi\upsilon^2}{c^3} < E >$$

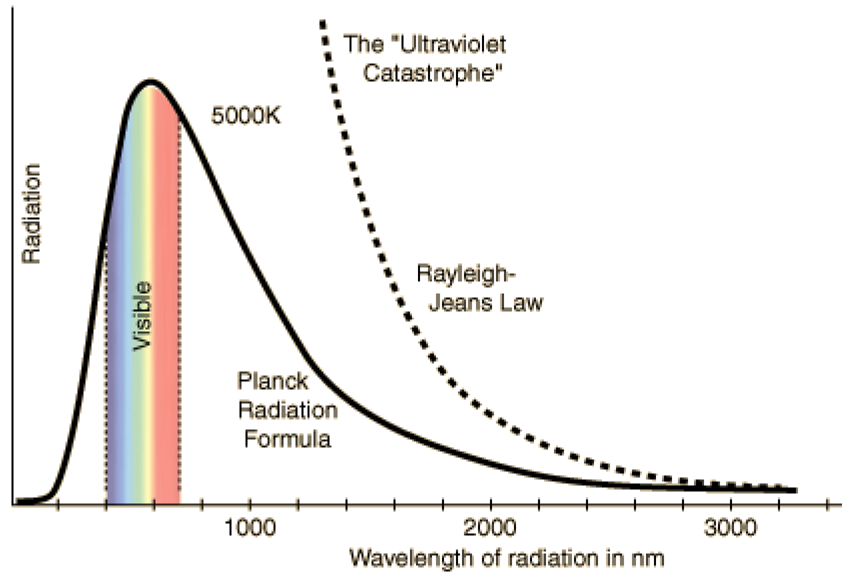
$$< E > = kT$$

Black body Radiation

- Although the Rayleigh-Jeans law works for low frequencies, it diverges at high v
- This divergence for high frequencies is called the ultraviolet catastrophe.



- Classical physics would predict that even relatively cool objects should radiate in the UV and visible regions. In fact, classical physics predicts that there would be no darkness!



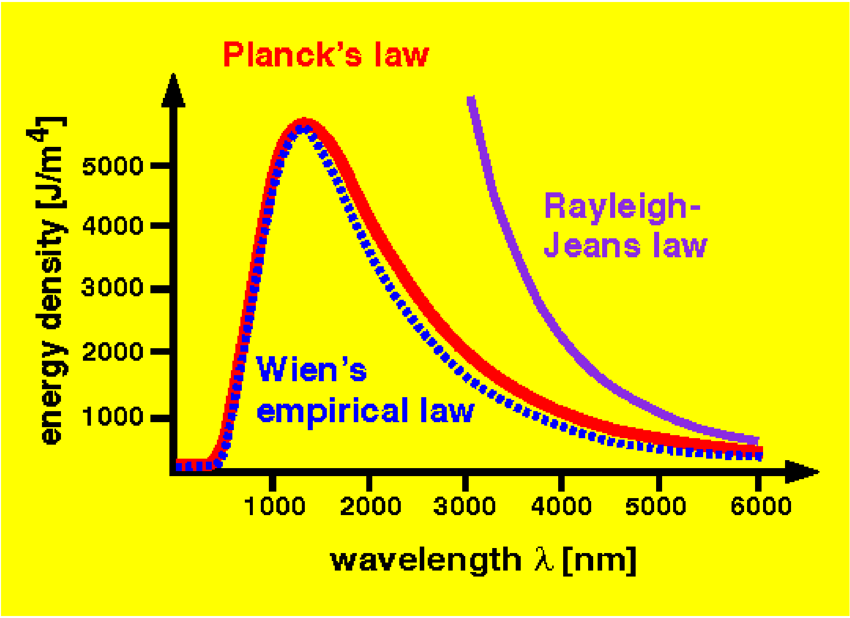
Black body Radiation

- Max Planck explained the blackbody radiation in 1900 by assuming that the energies of the oscillations of electrons which gave rise to the radiation must be proportional to integral multiples of the frequency, i.e., $E = nhv$

Planck's Concept

- The average energy per "mode" or "quantum" is the energy of the quantum times the probability that it will be occupied

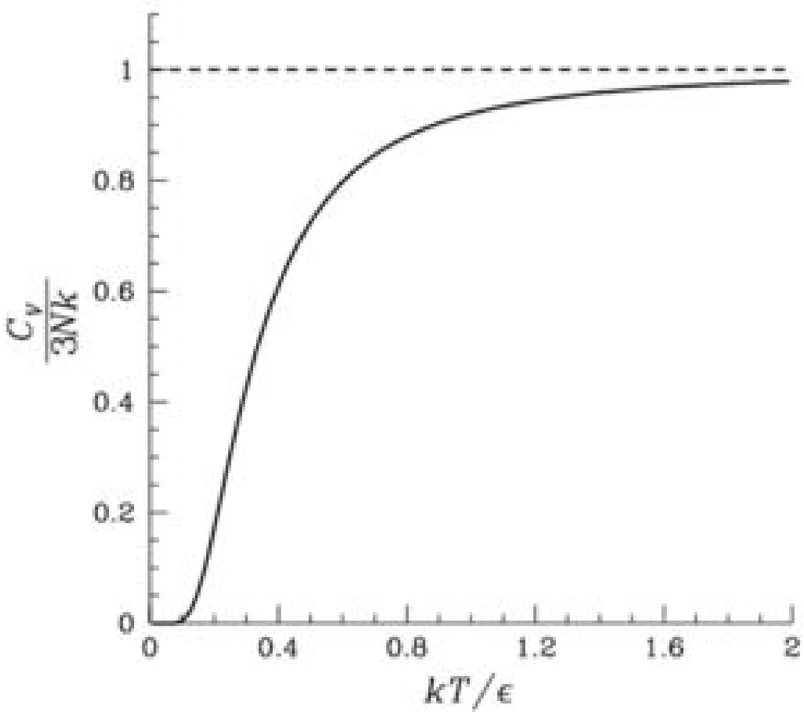
$$\langle E \rangle = \frac{h\upsilon}{e^{h\upsilon/kT} - 1}$$



- Planck won the Nobel Prize in Physics in 1918

The recognition that energy changes in discreet quanta at the atomic level marked the beginning of quantum mechanics.

Heat capacities of monoatomic solids



- Classical physics predicts a constant value (25 $JK^{-1}mol^{-1}$) for the molar heat capacity of monoatomic solids.

- Experiments at low temperatures, however, revealed that the molar heat capacity approaches zero when temperature approaches zero.

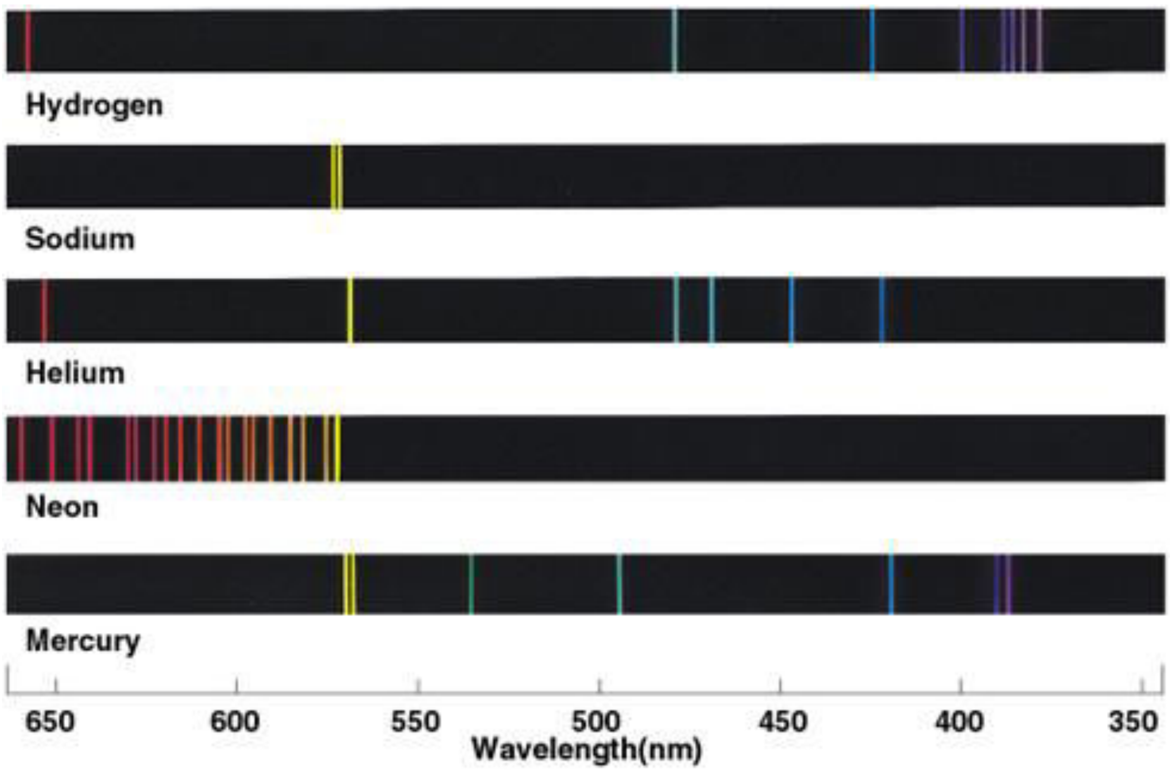
- Assumption of discrete energy levels (a collection of harmonic oscillators) again led to a model that matched the experimental observations (Einstein (1905)).

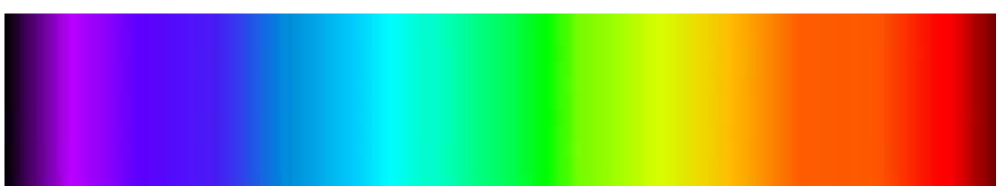
Atomic and molecular spectra

- Atomic and molecular spectra: Absorption and emission of electromagnetic radiation (i.e., photons) by atoms and molecules occur only at discrete energy values.
- Classical physics would predict absorption or emission at all energies.

Emission Lines

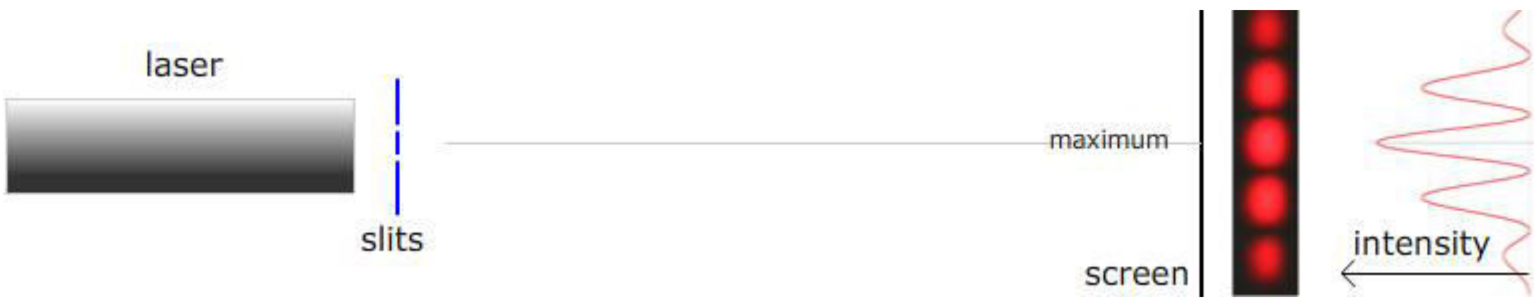
- Every element has a DIFFERENT finger print.





Wave or particle ??

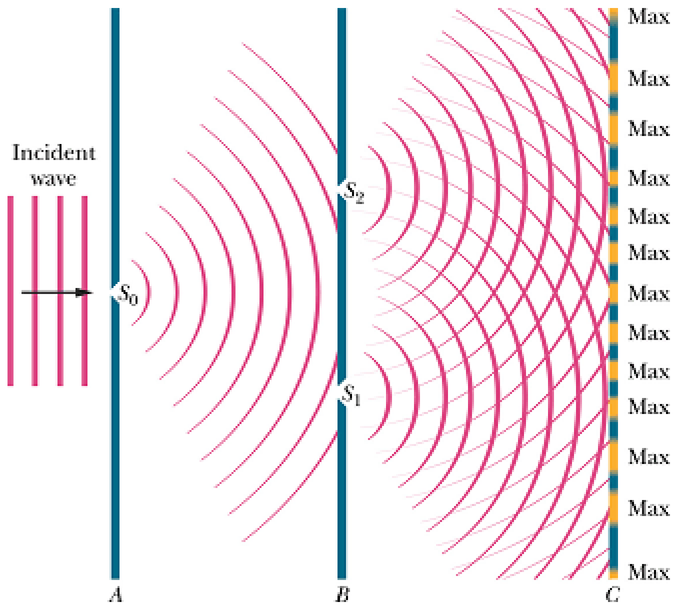
Young’s Double Slit experiment



A diffraction pattern of alternating dark and bright fringes

Young Double Slit Experiment

(Wave Nature of Light)



The **interference pattern would arise** only if we **consider electrons as waves**, which interfere with each other (i.e. constructive and deconstructive interference).

Wave nature of photon (De Broglie)

- Particles such as electrons has a wave description.

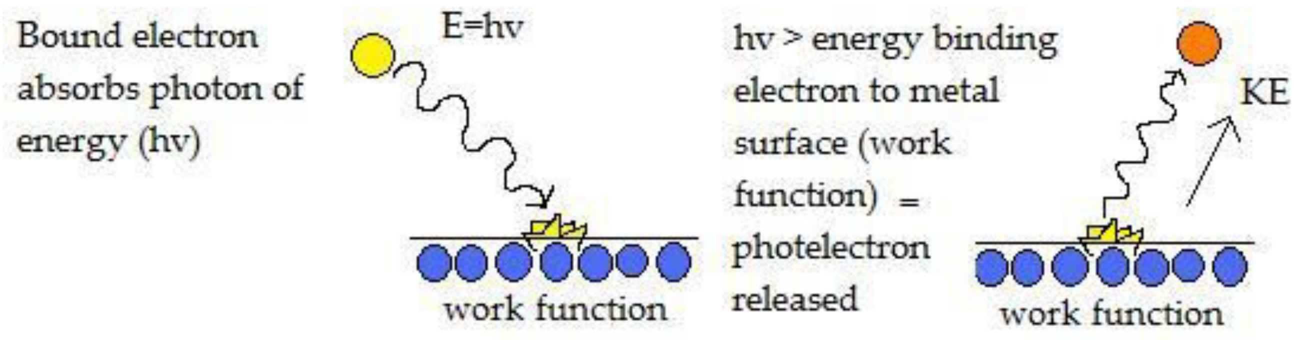
$$\lambda = \frac{h}{p}$$

De Broglie wave

- The de Broglie wave for a particle is made up of a superposition of an infinitely large number of waves of form

$$\psi(x,t) = A\sin 2\pi\left(\frac{x}{\lambda} - \upsilon t\right)$$

The waves that are added together have infinitesimally different wavelengths. This superposition of waves produces a wave packet.



RED LIGHT EJECTS SLOWER ELECTRONS THAN BLUE LIGHT

Photoelectric effect

- For a particular metal and a given color of light, say blue, it is found that the electrons come out with a well-defined speed, and that the number of electrons that come out depends on the intensity of the light.
- If the intensity of light is increased, more electrons come out, but each electron has the same speed, independent of the intensity of the light.
- If the color of light is changed to red, the electron speed is slower, and if the color is made redder and redder, the electrons’ speed is slower and slower.
- For red enough light, electrons cease to come out of the metal.

Failure of classical theory to explain Photoelectric effect

Using the classical Maxwell wave theory of light,

- **the more intense the incident light the greater the energy** with which the electrons should be ejected from the metal. That is, the average energy carried by an ejected (photoelectric) electron should increase with the intensity of the incident light.

Hence it is expected : **lag time** between exposure of the metal and emission of electron.

Therefore, classical theory could not explain the characteristics of photoelectric effect:

- Instantaneous emission of electrons
- Existence of threshold frequency
- Dependence of kinetic of the emitted electron on the frequency of light.

Einstein explanation of Photoelectric effect

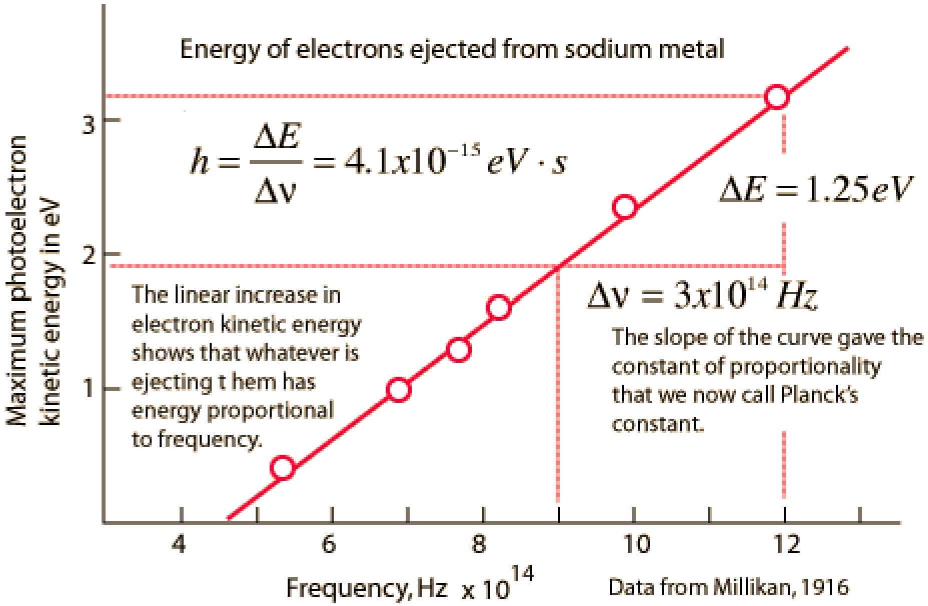
Einstein (1905) successfully resolved this paradox by employing Planck's idea of quantization of energy and proposed that the incident light consisted of individual quanta, called photons, that interacted with the electrons in the metal like discrete particles, rather than as continuous waves.

$h\nu = K.E + W$

where ν is frequency of radiation, K.E. is kinetic energy of emitted electron, W is work potential

$W = h\nu_0$ (ν_0 is threshold frequency)

$K.E. = h\nu - h\nu_0$
$K.E. = h(\nu - \nu_0)$



Though most commonly observed phenomena with light can be explained by waves. But the photoelectric effect suggested a **particle nature for light**.

Wave-Particle Duality: Light

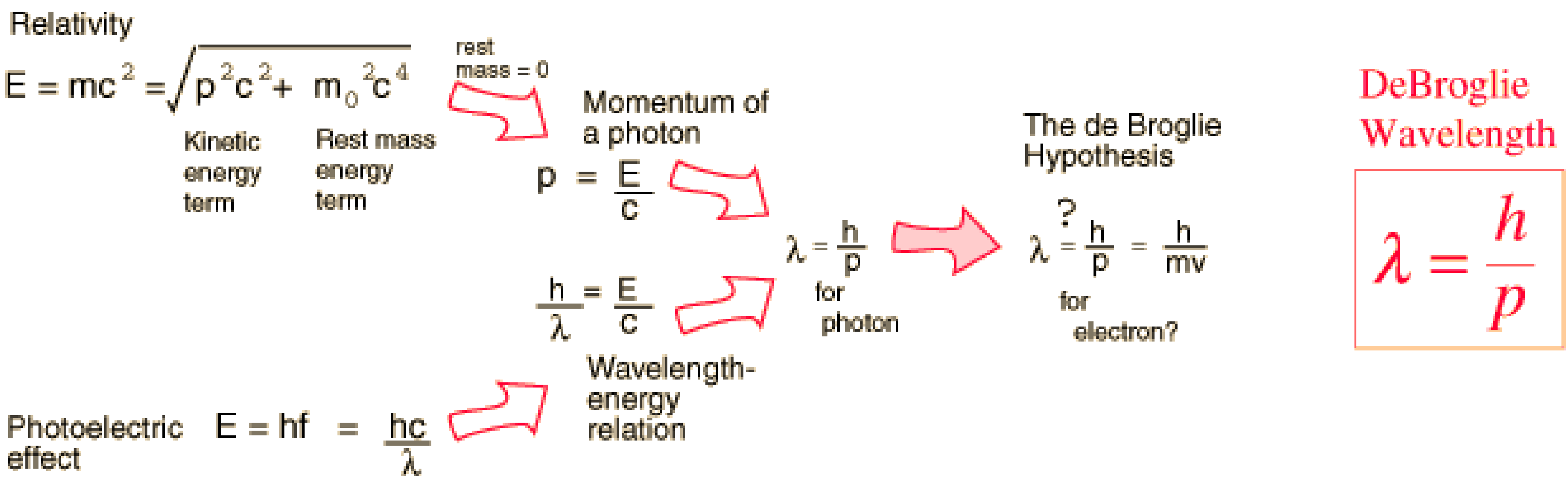
Does light consist of particles or waves? When one focuses upon the different types of phenomena observed with light, a strong case can be built for a wave picture:

Phenomenon	Can be explained in terms of waves.	Can be explained in terms of particles.
Reflection	✓	✓
Refraction	✓	✓
Interference	✓	⊗
Diffraction	✓	⊗
Polarization	✓	⊗
Photoelectric effect	⊗	✓
Compton scattering	⊗	✓

Most commonly observed phenomena with light can be explained by waves. But the photoelectric effect and the Compton scatering suggested a particle nature for light. Then electrons too were found to exhibit dual natures.

Wave Nature of Electron

As a young student at the University of Paris, Louis DeBroglie had been impacted by relativity and the photoelectric effect, both of which had been introduced in his lifetime. The photoelectric effect pointed to the particle properties of light, which had been considered to be a wave phenomenon. He wondered if electons and other "particles" might exhibit wave properties. The application of these two new ideas to light pointed to an interesting possibility:



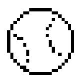
Confirmation of the DeBroglie hypothesis came in the **Davisson- Germer experiment** which showed interference patterns – in agreement with **DeBroglie wavelength** – for the scattering of electrons on nickel crystals.

DeBroglie Wavelengths

DeBroglie Wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

Does this relationship apply to all particles? Consider a pitched baseball:



$v = 40\ m/s = 90\ mi/hr$

$m = 0.15\ kg$

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ J\ s}{(0.15\ kg)(40\ m/s)} = 1.1 \times 10^{-34}\ m$$

$10^{-10} m$ Atomic diameter

$10^{-14} m$ Nuclear Diameter

For an electron accelerated through 100 Volts: $v = 5.9 \times 10^{6}\ m/s$

$$\lambda = \frac{6.626 \times 10^{-34} Js}{(9.11 \times 10^{-31} kg)(5.9 \times 10^{6}\ m/s)} = 1.2 \times 10^{-10} = 0.12\ nm$$

This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

- **The de Broglie wavelength λ for macroscopic particles are negligibly small**
- **This effect is extremely important for light particles, like electrons.**

Wave or Particle (Size ??)

- Objects that are large in the absolute sense have the property that the wavelengths associated with them are completely negligible compared to their size. Therefore, large particles only manifest their particle nature, they never manifest their wave nature.

Wave-particle duality and Heisenberg Uncertainty Principle

Can the radius be specified exactly ?

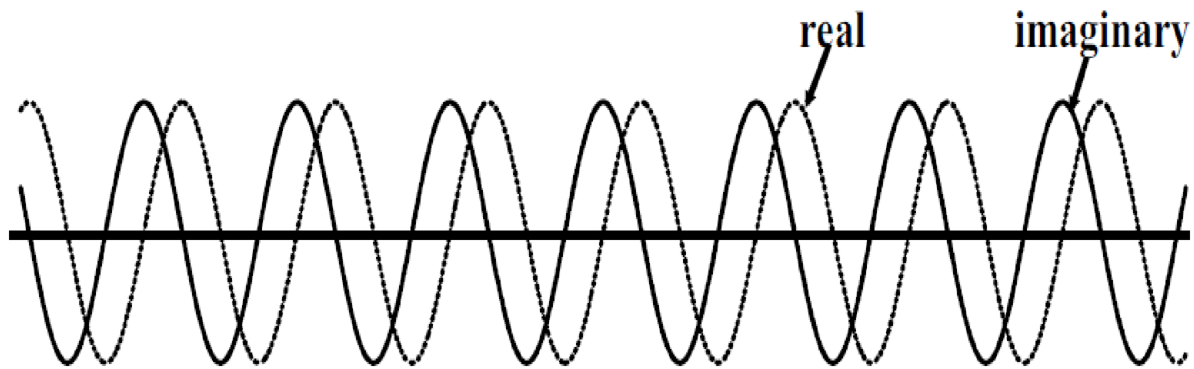


FIGURE 6.1. *The wavefunction for a free particle with momentum p, which has wavelength, $\lambda = h/p$. A quantum mechanical wavefunction can have two parts, called real and imaginary. Both waves have the same wavelength. They are just shifted by one-fourth of a wavelength, which is the same as a 90° shift in the phase. These two components are separate from each other. They do not interfere either constructively or destructively. For a free particle with the well-defined value of the momentum, p, the wave function extends from positive infinity to negative infinity, $+\infty$ to $-\infty$.*

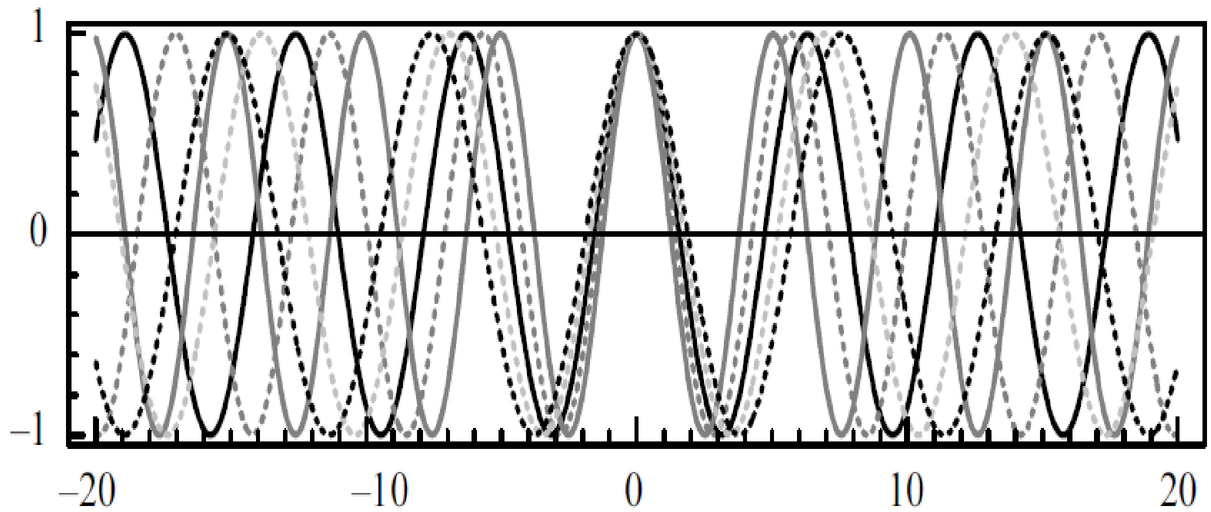


Figure 6.2. *Five waves are shown that have different wavelengths. The wavelengths are $\lambda = 1.2, 1.1, 1.0, 0.9$, and 0.8. The phases are adjusted so all of the peaks of the waves match at 0 on the horizontal axis. However, because the waves have different wavelengths, they do not match up at other positions, in contrast to Figure 3.2. Note that at a position of approximately 10 or -10, the dark gray wave has a positive peak, but the dashed light gray wave has a negative peak.*

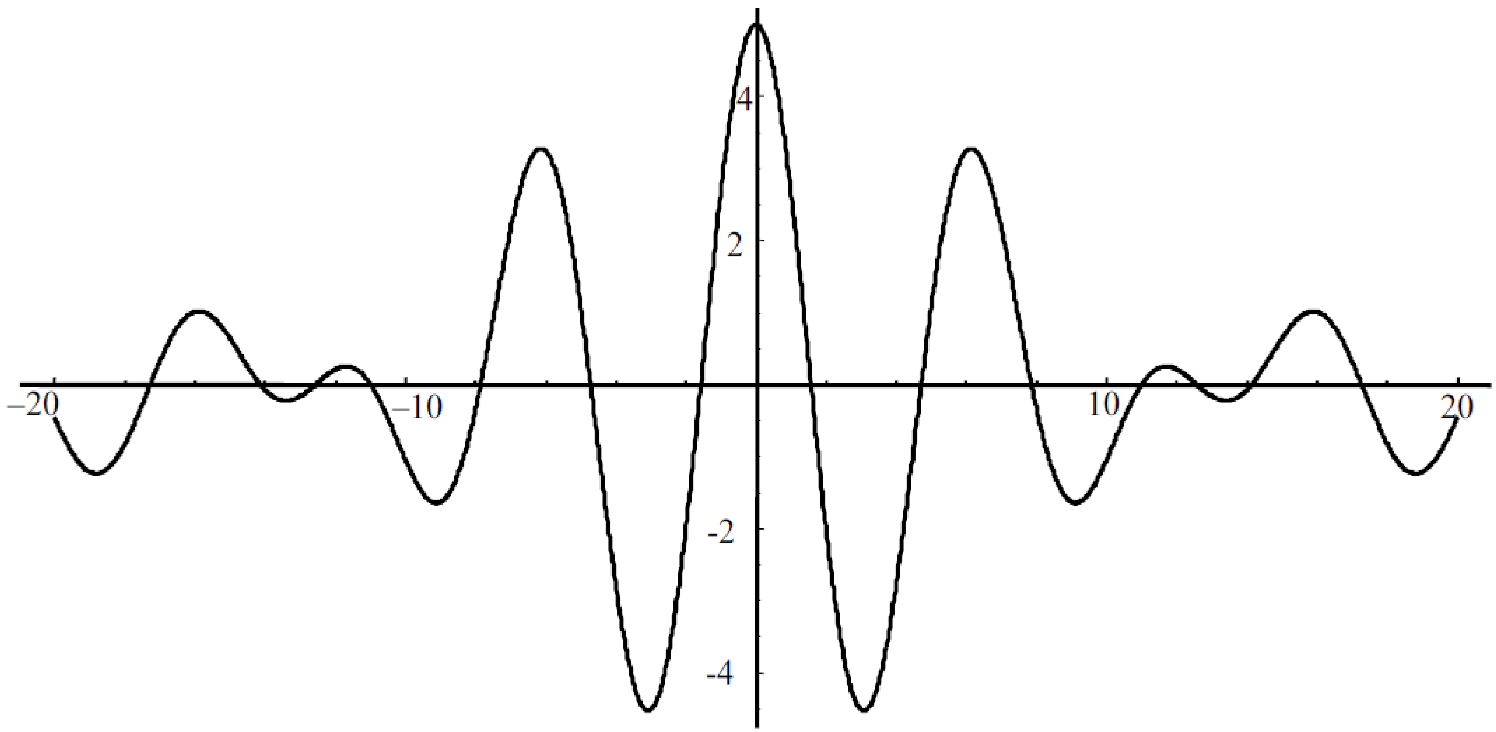


FIGURE 6.3. *The superposition of the five waves shown in Figure 6.2. At $x = 0$ (horizontal axis), all of the waves in figure 6.2 are in phase, so they add constructively. Near $x = 0$, the waves are still pretty much in phase, but the next set of maxima at about $x = 6$ and -6 are not as large as the maximum at $x = 0$. In the regions between 10 and 20 and -10 and -20, the difference in wavelengths makes some of the waves positive, where others are negative. There is significant cancellation.*

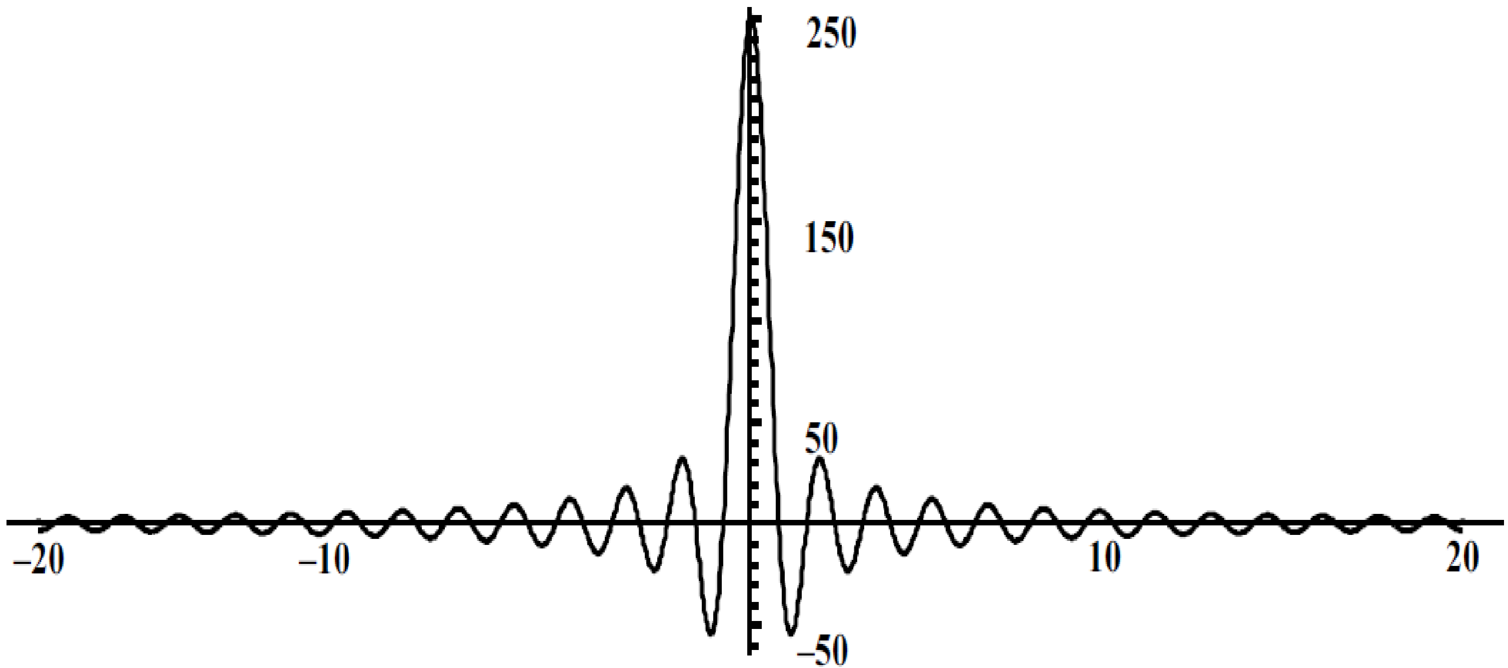


Figure 6.4. *The superimposition of 250 waves with equally spaced wavelengths spanning the wavelength range 0 to 4. Compared to Figure 6.3, which is the superposition of five waves, this superposition has a much larger peak at $x = 0$, the region of maximum constructive interference, and destructive interference reduces the other regions more. The amplitude of the superposition is dying out going toward $+20$.*

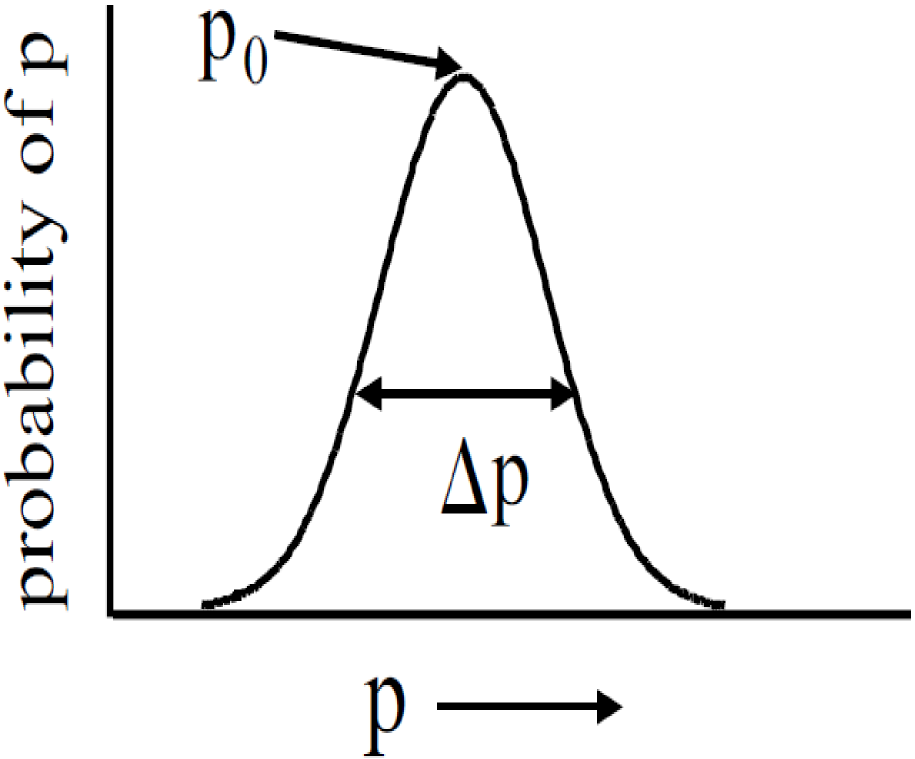


FIGURE 6.5. *A plot of the probability of finding a particle in a particular momentum eigenstate with momentum p given that it is in a superposition of momentum probability amplitude waves.* p_0 *is the middle wave with the biggest amplitude in the distribution.* Δp *is a measure of the width of the distribution of eigenstates.*

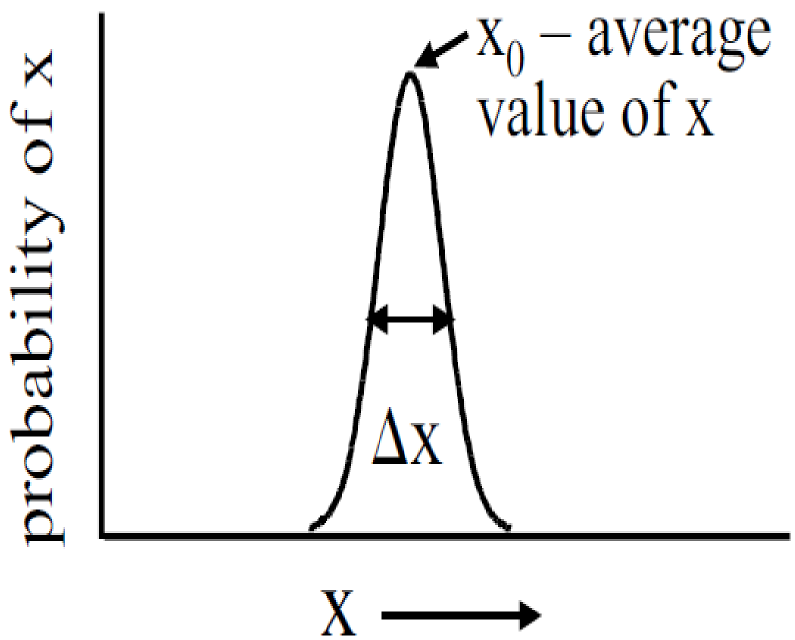


FIGURE 6.6. *A plot of the probability of finding the particle at a location x given that it is in the superposition of momentum eigenstates shown in Figure 6.5. x_0 is the middle position with the greatest probability. Δx is a measure of the width of the spatial distribution.*

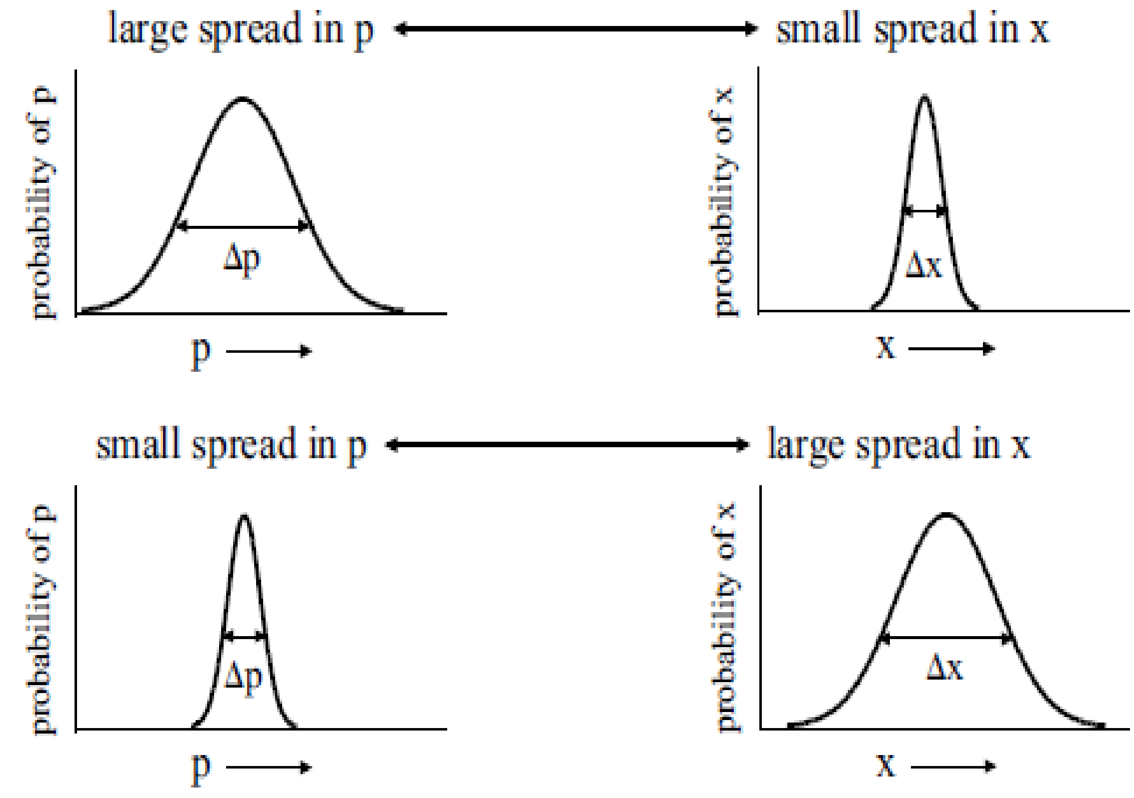


FIGURE 6.7. *The momentum (p) probability distributions and position (x) probability distributions for two wave packets. At the top, there is a large spread p (large Δp), which produces a small spread in x (small Δx). At the bottom, there is a small spread in p (small Δp), which gives rise to a large spread in x (large Δx).*